HYDRODYNAMICS AND HEAT TRANSFER INVOLVED IN THE BUBBLING OF A GAS THROUGH VISCOUS AND ANOMALOUS-VISCOUS LIQUIDS

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Results are shown of an experimental and theoretical study concerning the hydrodynamics and heat transfer involved in the bubbling of a gas through viscous and anomalous-viscous liquids with power-law rheological characteristics.

In the polymer industry it is often necessary to handle highly viscous liquids with non-Newtonian rheological characteristics. The heat transfer in such media has been studied to a limited extent only, even though serious problems are encountered here in attempts to make it more effective. Earlier studies by several authors [1-4, 10] have shown that the heat transfer rate can be increased considerably by bubbling a gas through the liquid. These studies were made, however, primarily with Newtonian and non-Newtonian liquids of low viscosity (dynamic viscosity not above 0.5 N · sec/m^2) [4].

Since during polymerization, solvent extraction, and similar processes it is necessary to supply or to extract huge quantities of heat, and since a reactor design with built-in extra surfaces is unreliable because of the difficulty in maintaining them clean, hence it may be worthwhile to explore the feasibility of heat transfer to the walls of a small-diameter bubble reactor.

We present here the results of studies concerning the hydrodynamics and the heat transfer involved in bubbling various gases through Newtonian and non-Newtonian liquids with power-law characteristics:

$$\tau = k\left(\frac{du}{dy}\right)^n, \tag{1}$$

where the dynamic viscosity was varied within the μ = 0.07-25 N · sec/m^2 range and the effective Prandtl number $\mathrm{Pr_{eff}}$ varied from 83 to $2.5 \cdot 10^5$.

In our experiment air was bubbled through aqueous solutions of sodium carboxymethylcellulose (Na-CMC) of various concentrations (c = 7.5-13.5%) and ethylene was bubbled through benzene solutions of polyethylene. Since polyethylene dissolves in benzene at elevated temperatures (150-160°C), moreover, the tests were performed under a pressure of approximately 0.9 MN/m^2.

In order to generalize and present the data in terms of dimensionless relations, we measured the rheological characteristics of the solutions with both a capillary viscometer and a Hepler rheoviscometer.

Na-CMC solutions follow a power law according to (1). Polyethylene solutions in our test range of shearing rates have slightly anomalous characteristics.

The thermophysical properties of Na-CMC solutions were determined on the basis of data for water, and those of polyethylene solutions in benzene were determined with the aid of a cylindrical $\alpha-\lambda$ calorimeter. All data pertaining to these solutions are given here next to the respective graphs.

The Na-CMC solutions were tested in an apparatus consisting of a vertical column, with an inside diameter 0.13 m and height 3 m, heated electrically by a coil element. In the middle of this column was installed a core 0.03 m in diameter. In this way, the cylindrical gap between the outer tube and the inner core was 0.05 m wide.

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The upper part of the column was passed through the bottom of a separator and the segment protruding coaxially inside the latter was capped with a deflector. The separator served to keep splashes of a solution away from air and condensed water vapor. For this purpose, a coil in a jacket was built into it through which cooling water could be pumped. The inside diameter of the separator was 0.4 m.

Three windows for visual inspection were provided at different heights in the side wall of the separator, making it possible to observe the solution level in the accelerating zone of the cylindrical gap between the inner core and the outer shell.

The test solution was circulated through the closed system: column–separator–pump–column.

The average gas content in the column was determined, with a mean error of ±5%, by measuring the volume (level) of the solution in the separator before and after air injection.

The heat transfer was studied in a cylindrical test segment 0.5 m long and placed 2 m above the air inlet, where the wall temperature was measured at three locations, the temperature of the solution was measured at the entrance to and at the exit from that test segment, and where also the electric power in the heater coil around the tube was measured.

The water vapor which the air carried away from the upper part of the separator to the exhaust into the atmosphere was condensing on the coil mounted inside the separator as well as on the water-cooled separator wall and flowing back into the solution. This ensured a constant concentration of the circulating solution, which was checked by viscosity measurements on samples.

The solution flowed through the column at a constant velocity of 0.025 m/sec. According to control measurements of the gas content and the heat transfer, the flow velocity of a circulating solution within the 0-0.037 m/sec range should have almost no effect on either the gas content or the heat transfer inside the column and, therefore, the conditions in the column could be considered to correspond to a zero mean velocity of the liquid phase – in agreement also with the results in [10].

The heat transfer in benzene solutions of polyethylene was studied in an apparatus consisting of a test channel, a separator, and appurtenances.

The test segment comprised a vertical column, inside diameter 0.2 m and height 3 m, with a core 0.1 m in diameter mounted inside. Thus, the cylindrical gap between the inner core and the outer tube of the column was 0.05 m wide. Both the column and the core were heated with steam trapped in steam pockets after passing the separator; the latter serving to keep away droplets of condensate coming from the steam main.

The pressure in this steam-heating system was automatically maintained constant at the necessary level.

The operating pressure (~0.9 MN/m^2) in the test gap was stabilized by means of an automatic regulating system.

A prepared solution of polyethylene in benzene was poured into the lower part of the column simultaneously with ethylene preheated to the temperature of the solution ($t = 160°$). The solution and the ethylene were flowing upward through the cylindrical gap. At this stage the ethylene was becoming saturated with benzene vapor.

A concentrated polyethylene solution and ethylene saturated with benzene vapor were passing together from the upper part of the evaporator into the separator.

From the separator, where splashes of the polyethylene solution were cleared of ethylene and benzene vapor, that solution was passing to the condenser. Dense solution from the separator was continually further processed.

Along the height of the cylindrical gap were installed five thermocouples and a few connecting tubes, for measuring the temperature inside this channel and for drawing samples of the solution without distorting the operating mode of the apparatus.

The thermal flux and the heat transfer coefficient were determined by measuring the quantity of condensate and the quantity of uncondensed vapor. Heat losses were tared out by heating the empty apparatus.

Since the flow pattern of a gas–liquid mixture is very complex, we will first present the test data and then arrive at possible semiempirical generalizations.

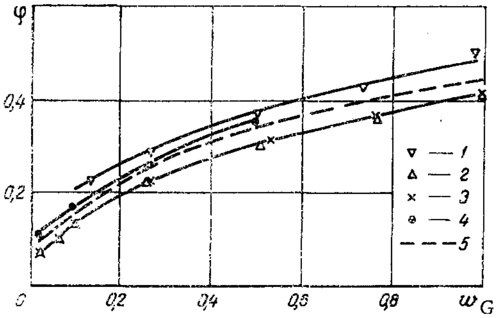

Fig. 1. Gas content as a function of the referred air velocity, when air is bubbling through water and aqueous solutions of Na-CMC at various concentrations: 1) pure water; 2) 7.5% Na-CMC; 3) 10%; 4) 13.5% (μ_{eff} = 0.001-6.0 N $\cdot$ sec/m^2); 5) according to formula (2). Air velocity w_G (m/sec).

Gas Content. The gas content is an important operating parameter, especially in the case of reactors and evaporators, when the dwell time of a solution in the apparatus must be known. The gas content in bubble apparatus has been dealt with in several references [5, 6, etc.].

No test data are available, however, for liquids as viscous as those in our experiment. There is also a lack of information about mixed flow of gas and a high-viscosity liquid through pipes or channels.

Our data in Fig. 1 indicate that the gas content is almost independent of the viscosity as the latter varies from 0.001 to 6.0 N $\cdot$ sec/m^2.

This result does not contradict the known physical concepts about the structure of gas–liquid streams in narrow pipes and channels, where in a viscous medium the bubbles usually buoy to the surface in a ballistic mode [5].

Considering that also other physical properties of the liquid have little influence on the gas content [5] and that φ itself appears in the heat transfer equation in a fractional-power term, the test data in Fig. 1 will fit into the following empirical relation:

$$\frac{\varphi}{1-\varphi}=\frac{2}{3}\left(\frac{w_G}{\sqrt{g\delta}}\right)^{2/3} \tag{2}$$

(curve 5 in Fig. 1) and this relation can, evidently, be extended also to higher pressures (0.5-1.0 MN/m^2).

Heat Transfer to the Wall. The results of heat transfer tests are shown in Fig. 2a, b; for the Na-CMC solutions (a) and for the polyethylene solutions in benzene (b).

According to Fig. 2a, b the heat transfer coefficient decreases as either the concentration or the viscosity increases. The absolute values of the heat transfer coefficient are, however, much higher than without bubbling.

In the heat transfer tests with polyethylene solutions in benzene the concentration of polyethylene varied from 8-12% at the entrance to 15-30% at the exit, because part of the benzene had evaporated along the way. Calculations and special tests (sampling the solution along the apparatus height) have shown that in this case the concentration of polyethylene in the solution varied in the apparatus as a linear function of the height:

$$c = c_0 + \Phi\,\frac{x}{l}\,,$$

with c_0 denoting the initial concentration, l denoting the length of the heat transfer segment, x denoting the coordinate, and Φ being a constant.

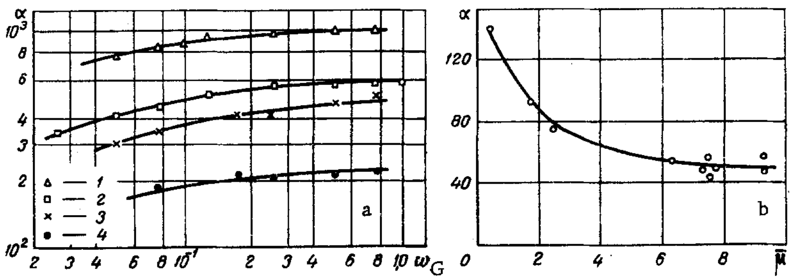

Fig. 2. Heat transfer coefficient as a function of the referred air velocity for Na-CMC solutions (a) [1) n = 0.725, k = 0.235; 2) n = 0.725, k = 1.14; 3) n = 0.70, k = 1.85; 4) n = 0.57, k = 12] and as a function of the mean viscosity of polyethylene solutions in the column for the referred gas (ethylene) velocities (w_G ≈ 0.07 m/sec) (b).

The rheological data (viscosity of the polyethylene solution) with the 8-30% range of concentration can be calculated by the formula:

$$\mu = A \exp kc \tag{3}$$

(based on the evaluation of test results) and from this, after a few simple transformations, we obtain for the mean viscosity:

$$\mu = \frac{\mu_c - \mu_0}{\ln \dfrac{\mu_c}{\mu_0}}. \tag{4}$$

The data shown in Fig. 2b are based on the mean viscosity (4) at a mean gas velocity of ~0.07 m/sec.

In order to generalize the heat transfer data, we will briefly review the status of the problem and then perform an approximate theoretical analysis.

According to many published sources [2, 7, 10, 11, etc.], during bubbling in vertical channels the rate of heat transfer from the wall does not depend on the channel geometry, at equal referred gas velocities, which makes it easier to simulate a bubble apparatus.

For low- and medium-viscosity liquids, up to 0.5-1.0 $\mathrm{N \cdot sec/m^2}$, the theoretical analysis in [3] and the experimental studies in [4] have yielded the following relation describing the heat transfer coefficient:

$$\frac{\alpha}{\lambda}\left(\frac{\nu^2}{g}\right)^{1/3} = Z\varphi^{1/3}\left(\frac{\nu}{a}\right)^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14}. \tag{5}$$

This equation may be used for calculating the heat transfer to non-Newtonian liquids, if the variation of the dynamic viscosity as a function of the referred gas velocity is also taken into account.

The derivation of (5) was based on a fully developed turbulence, with the following relation between the Nusselt number and the Reynolds number:

$$\mathrm{Nu} = Z\,\mathrm{Re}^n\,\mathrm{Pr}^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14}, \tag{6}$$

where $n = 0.67\text{-}0.70$.

This relation is not applicable to highly viscous liquids, because then

$$\mathrm{Nu} = Z\,\mathrm{Re}^{0.5}\,\mathrm{Pr}^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14}. \tag{7}$$

At the same time, because of the complex stream pattern, it may be advisable to apply here the concepts developed in [3, 7] and, namely, to determine the convection rate on the basis of energy dissipation.

For a viscous dissipation of energy we have from [8]:

$$\varepsilon = A\nu\rho\left(\frac{u_c}{l}\right)^2, \tag{8}$$

where the value of the constant A is 15-30 and $l = \delta(1-\varphi)$.

On the other hand, for the energy dissipation in a gas–liquid stream one can also write

$$\varepsilon = w_G(1-\varphi)\,\gamma_L. \tag{9}$$

Thus,

$$u_c = \delta\left(\frac{w_G(1-\varphi)^3 g}{A\nu}\right)^{1/2}. \tag{10}$$

Inserting this expression into (7) and letting the value of coefficient Z be the same as for the front segment of a plate [9] or pipe in a longitudinal stream, we obtain

$$\frac{\alpha\delta}{\lambda} = 0.33\left[\delta^2(1-\varphi)\left(\frac{w_G g}{A\nu}\right)^{1/2}\right]^{1/2}\mathrm{Pr}^{1/3}, \tag{11}$$

and, after simple transformations,

$$\frac{\alpha\delta}{\lambda}\left(\frac{\nu^2}{g}\right)^{1/3} = \frac{0.33}{A^{1/4}}\left[\frac{w_G}{(\nu g)^{1/3}}\right]^{1/4}(1-\varphi)^{1/2}\,\mathrm{Pr}^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14}. \tag{12}$$

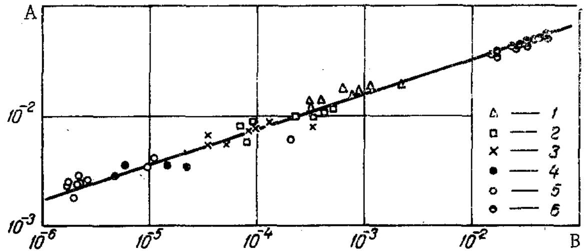

Fig. 3. Universal relation for determining the heat transfer coefficient. The symbols correspond to those in Fig. 2a, b. Test points 6 taken from [10]. A = $(\alpha/\lambda)(a^2/g)^{1/3}(\mu_W/\mu)^{0.14}$; B = $(a/\nu_{eff})[w_G/g\nu_{eff})^{1/3}]^{3/4}[1-(w_G^2/g\delta)^{1/4}]^{3/2}$.

A similar relation was derived in [7] for fully developed turbulence.

A comparison with the test data (Fig. 3; 1, 6) indicates a close agreement with expression (12), even in the case of low-viscosity liquids.

In order to generalize the heat transfer data over both Newtonian and non-Newtonian liquids, we rewrite expression (12) as

$$\frac{\alpha}{\lambda}\left(\frac{a^2}{g}\right)^{1/3}=Z_1\left[\frac{w_G}{(\nu_{eff}g)^{1/3}}\cdot\frac{a^{4/3}}{\nu_{eff}^{4/3}}\right]^{1/4}(1-\varphi)^{1/2}\left(\frac{\mu}{\mu_w}\right)^{0.14} \tag{13}$$

and find how the dynamic viscosity is related to the operating parameters. It is worthwhile here to define this relation also in terms of energy dissipation. For this we rewrite expression (8) as

$$\varepsilon=A\frac{\tau^2}{\mu_{eff}}. \tag{14}$$

From expression (1) we can easily obtain

$$\mu_{eff}=k^{1/n}\tau^{1-1/n}, \tag{15}$$

and, after a few transformations,

$$\mu_{eff}=k^{\frac{2}{n+1}}\left(\frac{\varepsilon}{A}\right)^{\frac{n-1}{n+1}}. \tag{16}$$

Considering also (9), we finally have

$$\nu_{eff}=\frac{k^{\frac{2}{n+1}}\left[w_G\gamma_L(1-\varphi)\frac{1}{A}\right]^{\frac{n-1}{n+1}}}{\rho}. \tag{17}$$

The final version of the test data is shown in Fig. 3, according to which Eq. (13) tentatively used for calculating the dynamic viscosity (17) applies to Newtonian as well as to non-Newtonian liquids.

In Fig. 3 are also shown the test data from [10] evaluated in terms of Eq. (13), which those authors had obtained in their tests with air bubbling at referred velocities w_G = 0.1-1.0 m/sec through aqueous solutions of 71% glycerin through pipes 0.032 m in diameter. It is evident from Fig. 3 that these data are closely approximated by Eq. (13).

It is to be noted that the presence of the $(1-\varphi)^{1/2}$ term in Eq. (13) explains why the heat transfer rate almost ceases to increase at a referred gas velocity within 0.2-0.4 m/sec.

Finally, with expression (2) and the test data taken into account, the expression for determining the heat transfer coefficient becomes

$$\frac{\alpha}{\lambda}\left(\frac{a^2}{g}\right)^{1/3}=0.155\left[\frac{w_G}{(g\nu_{eff})^{1/3}}\right]^{1/4}\left[1-\left(\frac{w_G^2}{g\delta}\right)^{1/4}\right]^{1/2}\left(\frac{a}{\nu_{eff}}\right)^{1/3}\left(\frac{\mu}{\mu_w}\right)^{0.14}. \tag{18}$$

Theoretically, constant Z_1 = 0.14-0.17 and constant A = 15-30.

As a result of an experimental and theoretical study of the hydrodynamics and the heat transfer, based on rheological, thermophysical, and thermodynamic data, we have thus established the relations describing the heat transfer process which occurs when gas is bubbling through viscous and anomalous-viscous liquids with the Prandtl number Pr_{eff} = 83 to $25 \cdot 10^4$.

NOTATION

w_G is the referred velocity of gas, m/sec;
u_c is the convection rate, m/sec;
τ is the shearing stress, N/m^2;
δ is the width of the cylindrical gap between the inner and outer tube, m;

α	is the heat transfer coefficient, $W/m^2 \cdot °C$;
φ	is the gas content;
n, k	are the rheological constants;
μ_{eff}	is the dynamic viscosity, $N \cdot sec/m^2$;
a	is the thermal diffusivity, m^2/sec;
ν	is the kinematic viscosity, m^2/sec;
λ	is the thermal conductivity, $W/m \cdot °C$;
g	is the acceleration of gravity, m/sec^2;
ρ	is the density of liquid, kg/m^3;
Z, A	are constants.

Subscripts

0	denotes the channel entrance;
K	denotes the channel exit;
L	denotes the liquid;
G	denotes the gas (air);
w	denotes the wall.

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